MATHEMATICS AND STATISTICS 2021 RESEARCH COMPETITION.

EXPONENTIAL GROWTH OF FRUIT FLIES GIVEN THE NUMBER OF WEEKS

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ABSTRACT

This report is a thorough statistical analysis on the population growth of fruit flies (Drosophilia melanogaster) that looks into research questions discussing both theoretical and actual statistics and factors as to why the two are so drastically different. It also examines the mathematical relationship between the statistical data and the progression of time (number of weeks) and explores this with consideration to the different stages of a fruit fly's life via researching, referring to secondary sources, and applying mathematical and technological methods.

Contents:

Introduction:

The aim of this task is to investigate, using mathematical methods, the growth of fruit flies in a standard fruit fly's life cycle and analyse the trends found in both the theoretical and actual data and compare the two sets of data using a mathematical model. As well as producing a rule representing the theoretical growth of the population and exploring the inaccuracies of this data due to the factors involved in a real life situation with different external factors impacting the results produced.

Assumptions/Approach:

The lifecycle of a fruit fly can be depicted below:

Egg → Larva → Pupa → Adult → Death

1 Week (Egg to Pupa) ***1 Week*** (Pupa to Adult) ***7 Weeks*** (Adult to Death)

For this investigation it can be assumed that it takes one whole week for each fruit fly to transition from 'egg' to 'pupa' and that it takes another week for the transition from 'pupa' to 'adult'. An average adult is expected to live for a span of 7 weeks before death and each male/female pair of adult fruit flies produces 10 eggs for each week they live (a total of 70 eggs across the average lifespan of an adult fruit fly), and that the total population of fruit flies will always maintain an equal number of male and female fruit flies (of the 10 eggs, 5 are expected to be male and the other 5 are expected to be female).

This investigation will be approached by creating a table of data that utilises the above rules to calculate the number of adult fruit flies present each week and then using technological methods and prior knowledge of exponential functions, find a rule that directly connects the number of weeks (the x-value on a graph) and the number of adult fruit flies at the end of each week (the y-value on a graph).

Table of Data:

The following table of data keeps track of the number of fruit flies in their various stages if a researcher were to start with one male/female pair of fruit flies in their pupa stage.

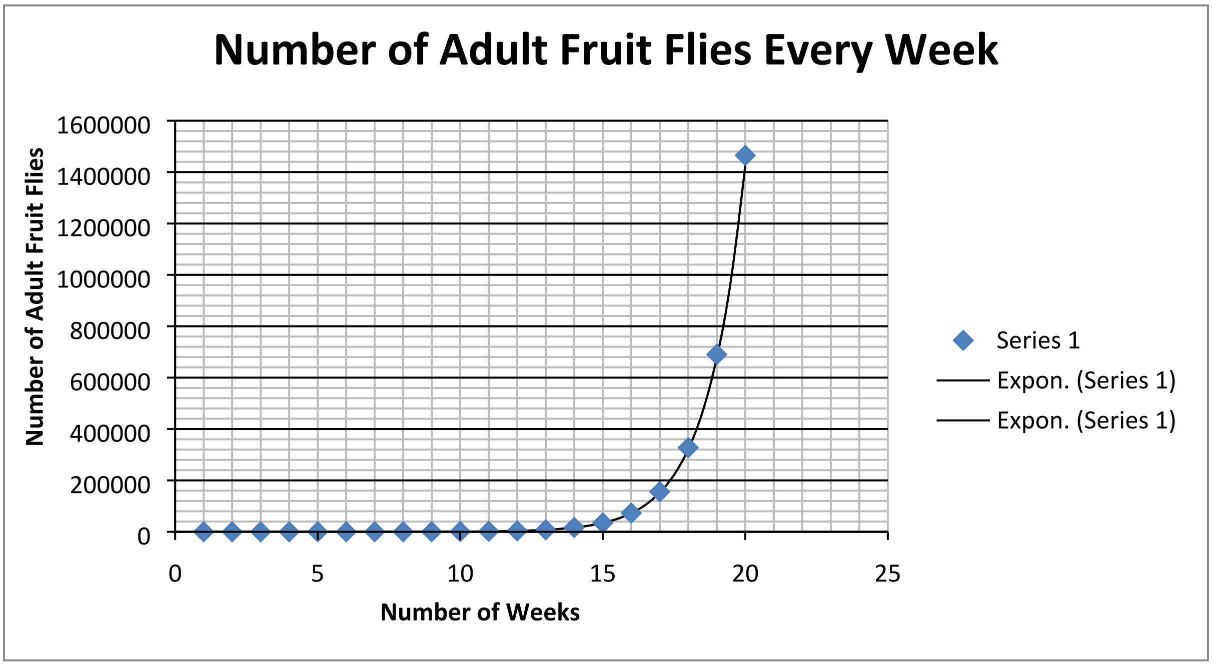
*M stands for male fruit flies while F stands for female fruit flies.

*The weeks are counted starting from the transition of the first pair of adult fruit flies.

Weeks	Eggs	Pupa	Adults
0	0	1M + 1F	0
1	0	0	1M + 1F
2	5M + 5F	0	1M + 1F
3	5M + 5F	5M + 5F	1M + 1F
4	5M + 5F	5M + 5F	6M + 6F
5	30M + 30F	5M + 5F	11M + 11F
6	55M + 55F	30M + 30F	16M + 16F
7	80M + 80F	55M + 55F	46M + 46F
8	230M + 230F	80M + 80F	100M + 100F

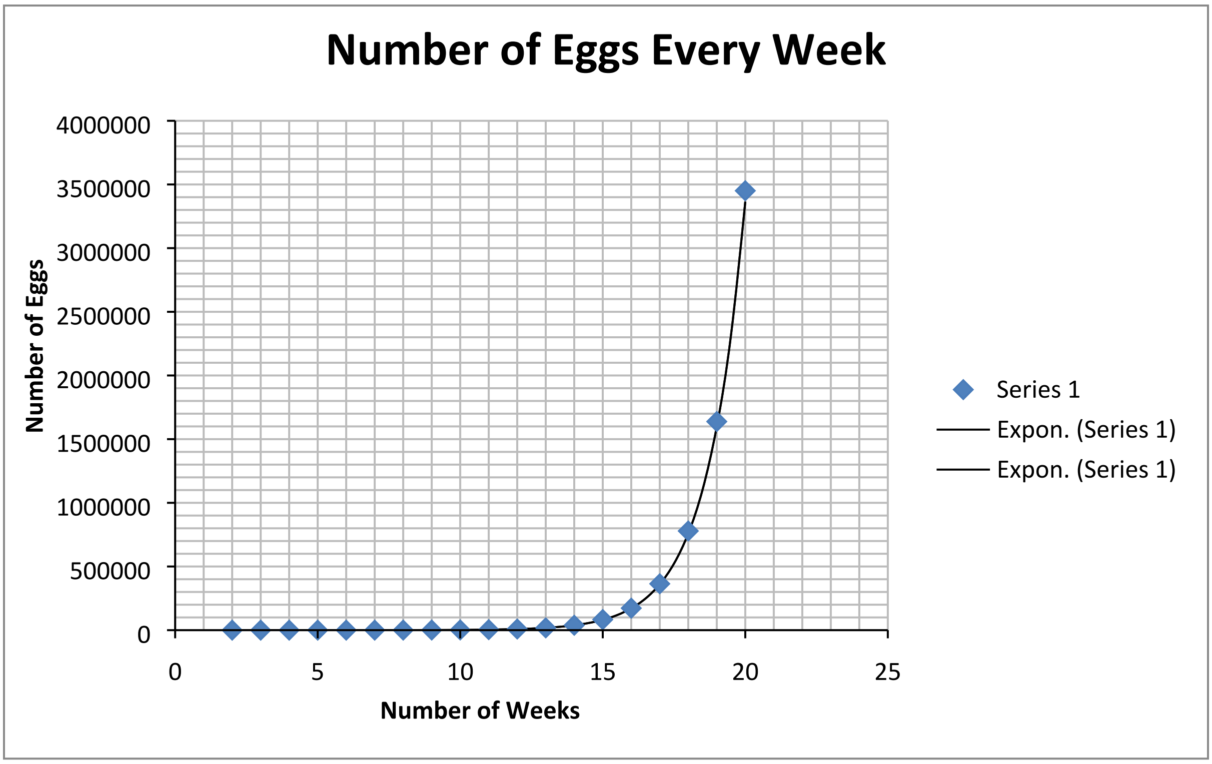
9	500M + 500F	230M + 230F	180M + 180F
10	900M + 900F	500M + 500F	410M + 410F
11	2050M + 2050F	900M + 900F	905M + 905F
12	4525M + 4525F	2050M + 2050F	1800M + 1800F
13	9000M + 9000F	4525M + 4525F	3845M + 3845F
14	19225M + 19225F	9000M + 9000F	8340M + 8340F
15	41700M + 41700F	19225M + 19225F	17285M + 17285F
16	86425M + 86425F	41700M + 41700F	36430M + 36430F
17	182150M + 182150F	86425M + 86425F	77900M + 77900F
18	389500M + 389500F	182150M + 182150F	163825M + 163825F
19	819125M + 819125F	389500M + 389500F	345075M + 345075F
20	1725375M + 1725375F	819125M + 819125F	732525M + 732525F

Mathematical Model/Graph:

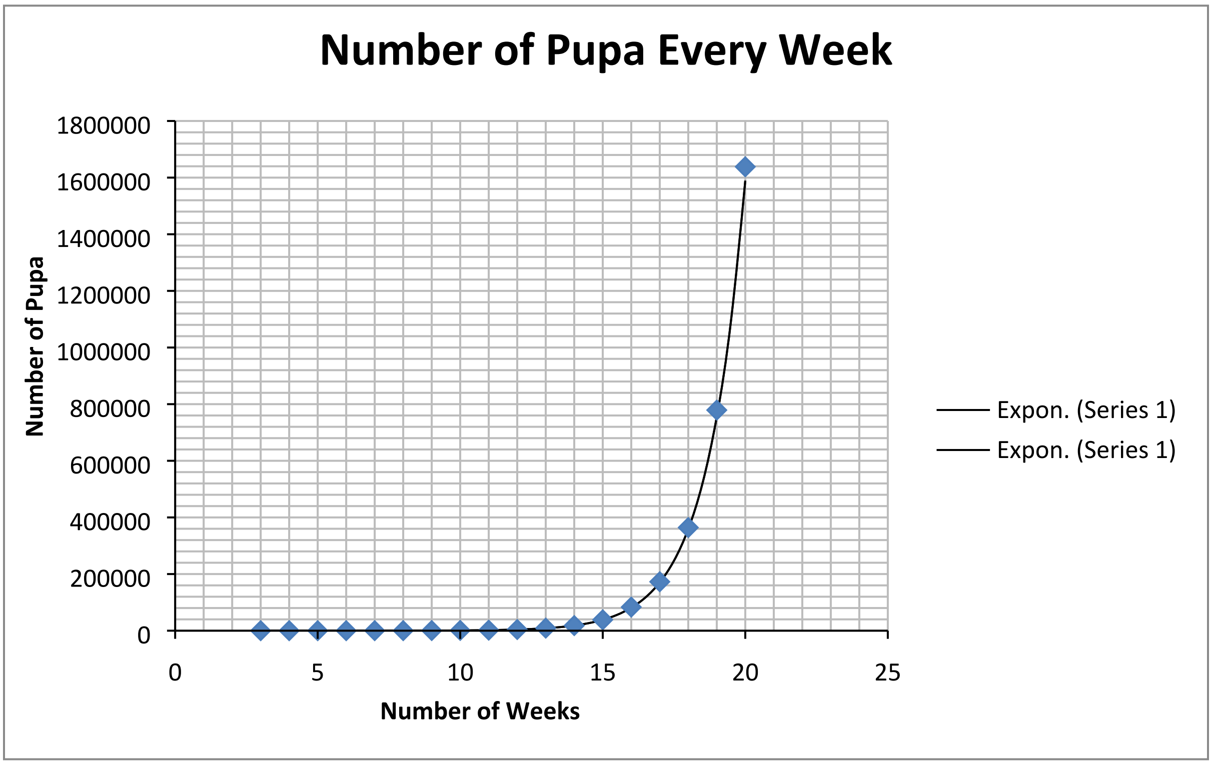


Relationship Between Number of Fruit Flies and Weeks:

As given from the graph above, an approximate rule for the relationship between the number of weeks and the number of adult fruit flies can be given by $\mathbf{y = 0.481e^{0.745x}}$. Other relationships relevant to the research topic include the relationship between the number of weeks and the number of eggs and the relationship between the number of weeks and the number of pupa, as is plotted on the graphs below. The graph depicting the relationship between the number of weeks and number of eggs is given by $\mathbf{y = 1.146e^{0.744x}}$and the relationship between the number of weeks and the number of pupa is given by $\mathbf{y = 0.546e^{0.744x}}$.

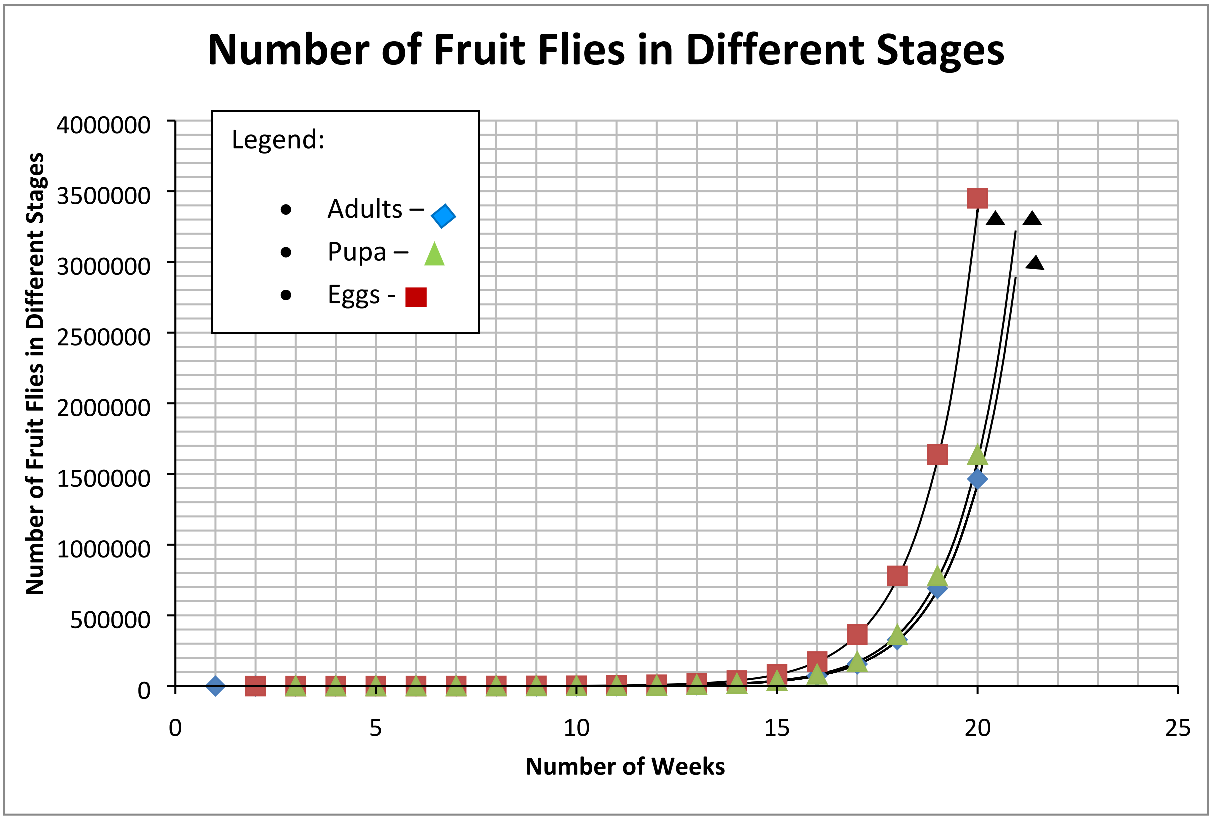


*The data above excludes any point at which there are zero eggs.



*The data above excludes any points at which there are zero pupas.

Comparing all three graphs and the rates of growth for each:



Data Analysis/Trends:

The equations for all the above graphs can be identified as differential equations. Differential equations are equations for functions that not only contain 'x' and 'y' variables but the rate of change of each of those variables so that the solution(s) to such equations are in themselves functions. Mathematically they relate one or more functions and their derivatives. Derivatives can be used to calculate the rate of change of the output values of the function relative to the rate of change of the input values of the function. To find the derivative of the function relating the number of adult fruit flies present to the number of weeks past, the following steps can be taken (where the derivative is given by the symbol$\frac{d}{dx}$) :

Step 1: Identifying Formulas - $\frac{d}{dx}[x^n] = nx^{(n-1)}$ and$\frac{d}{dx}[e^u] = \frac{d}{dx}[u] \cdot e^u$

Step 2: Applying - $\frac{d}{dx}[0.481e^{0.745x}] = \frac{d}{dx}[e^{0.745x}] \cdot 0.481$

Step 3: Solving - $\frac{d}{dx}[0.745x] = 0.745$

Step 4: Solution - $\frac{d}{dx}[0.481e^{0.745x}] = 0.745 \cdot 0.481e^{0.745x} \approx \boldsymbol{0.358e^{0.745x}}$

Step 5: This can also be modelled into a graph and through optimisation; we can trace the relation back to the original equation and use it to observe the rate of change.

The same can be applied for the other equations above relating the number of pupas to the number of weeks and the number of fruit fly eggs to the number of weeks as shown below:

Pupa vs. Weeks:

Step 1: Identifying Formulas - $\frac{d}{dx}[x^{n}] = nx^{(n-1)}$ and $\frac{d}{dx}[e^{u}] = \frac{d}{dx}[u] \cdot e^{u}$

Step 2: Applying - $\frac{d}{dx}[0.546e^{0.744x}] = \frac{d}{dx}[e^{0.744x}] \cdot 0.546$

Step 3: Solving - $\frac{d}{dx}[0.744x] = 0.744$

Step 4: Solution - $\frac{d}{dx}[0.546e^{0.744x}] = 0.744 \cdot 0.546e^{0.744x} \approx \boldsymbol{0.406e^{0.744x}}$

Step 5: This can also be modelled into a graph and through optimisation; we can trace the relation back to the original equation and use it to observe the rate of change.

Eggs vs. Weeks:

Step 1: Identifying Formulas - $\frac{d}{dx}[x^{n}] = nx^{(n-1)}$ and $\frac{d}{dx}[e^{u}] = \frac{d}{dx}[u] \cdot e^{u}$

Step 2: Applying - $\frac{d}{dx}[1.146e^{0.744x}] = \frac{d}{dx}[e^{0.744x}] \cdot 1.146$

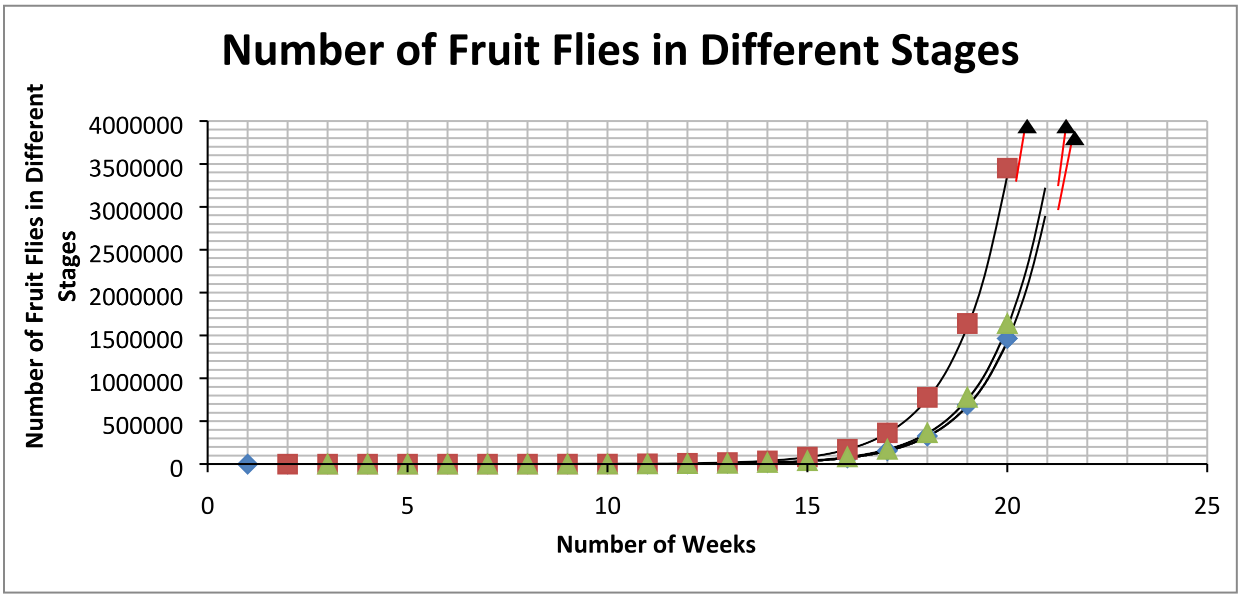
Step 3: Solving - $\frac{d}{dx}[0.744x] = 0.744$

Step 4: Solution - $\frac{d}{dx}[1.146e^{0.744x}] = 0.744 \cdot 1.146e^{0.744x} \approx \boldsymbol{0.853e^{0.744}}$

Step 5: This can also be modelled into a graph and through optimisation, we can trace the relation back to the original equation and use it to observe the rate of change.

Further trends regarding the exponential growth of the number of eggs, pupa and adult fruit flies suggests that theoretically this growth should be sustained and infinite and that the rate of reproduction among fruit flies is much bigger than the rate of deaths among fruit flies. For this reason it should theoretically be impossible for the number of deaths and the number of eggs produced to reach equilibrium. For instance from the 18^{th} to the 19^{th} week shown on the table of data, 1800 adult fruit flies die compared to the 1638250 eggs produced. The rate of death of the adult fruit flies in the table on average is approximately 0.6%, in that 0.6% of the previous week's adult fruit flies die, on the other hand the number of eggs produced every week grows exponentially at an average rate of 210%, in that the eggs produced every week are almost 2.1 times the number of eggs produced the previous week. Comparing the different graphs also indicates that the rate of growth of the number of eggs is identical to the rate of growth of the number of pupa except that the graph representing pupa against weeks is translated one unit to the right (starts its growth one week later than the eggs). The graph representing the number of adult fruit flies shows a dilated graph that has a slower rate of growth than the other two graphs.

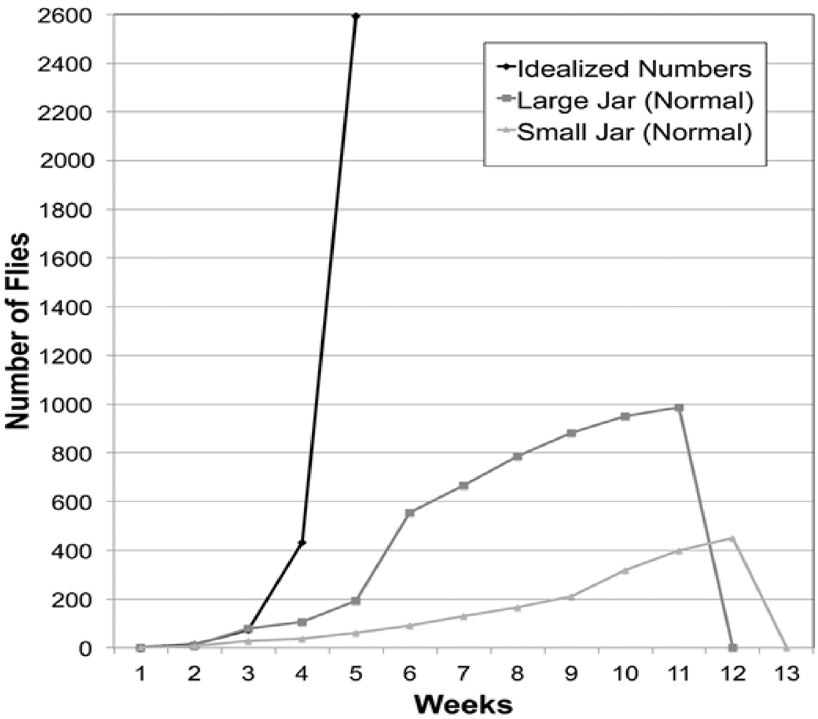
The data trends can be explored further by using the formulas produced to extrapolate the data to prove the hypothetical infinite growth of the fruit fly population, the following graphs show the trend line being extended a further week showing how dramatically the population is expanding and effectively predicting the next few points of data.



*The extrapolated parts are in red.

Comparing Theoretical Data to Actual Data

The theoretical data represented in the table of data is not an accurate representation of the growth of a real life population of fruit flies, the following secondary source (***APA in-text reference: Heaps; Dawson; Briggs; Hansen; Jensen, A., 2016. Deriving Population Growth Models by Growing Fruit Fly Colonies. [online] The American Biology Teacher. Available at: <https://online.ucpress.edu/abt/article/78/3/221/110077/Deriving-Population-Growth-Models-by-Growing-Fruit> [Accessed 8 July 2021].***)carried out a similar experiment to the one discussed in the investigation regarding the growth of the fruit fly population they were breeding and represented the resultant data in the following graph/mathematical model.



APA in-text reference: Heaps; Dawson; Briggs; Hansen; Jensen, A., 2016. Deriving Population Growth Models by Growing Fruit Fly Colonies. [online] The American Biology Teacher. Available at: <https://online.ucpress.edu/abt/article/78/3/221/110077/Deriving-Population-Growth-Models-by-Growing-Fruit> [Accessed 8 July 2021].

The above shows that the theoretical growth or idealised numbers of how the fruit fly population should grow does not at all match the numbers from a real life experiment, the fruit fly population grows at a slower rate and eventually reaches a point of decrease (which should theoretically be impossible) until the entire fruit fly population eventually dies out (also theoretically impossible). There are many possible contributing for this such as biological factors (eg. diseases both genetic and viral) and reproductive factors that make it impossible to determine or predict the growth of a population. The following discuss such factors in closer detail:

Climate and Location:

Drosophila Melanogaster, the common fruit fly, is flexible in the various climates they live in, they survive in both very warm and cold temperature, but they differently affect their mating cultures. They are present and thriving in all continent and islands, emphasizing their flexibility and ability to cope in various environments but differ, depending on the location, the rate of reproduction and size of the flies produced.

Cold Weather:

In colder climates, such as that which is apparent in the northern and southern poles, the mating behaviors demonstrated by the flies alter. The female flies become less likely to mate with their male counterpart, thus, drastically reducing number of eggs produced. Due to their comparatively short lifespan, the Drosophila Melanogaster tend to rely on evolution, which is influenced by epigenetics (a system which allows for the transfer of information between multiple generations), to reestablish their 'natural' birth rate rather than to travel to an area with more suitable weather.

Hot Weather:

In warmer climates, it is observed that fruit flies reproduce at a slightly faster rate than that of the norm which further highlights the polygamous nature off the fruit flies' mateship. Since one female fly can mate with various other males, the rate and number of offspring doesn't change as one female can lay a specific number of eggs which is constant whether it is fertilized by one or multiple males, however, due to the increased interest in mating, the males are shown to be more violent towards one another to eliminate other male candidates which express interest in mating the female. Hence, many more male flies die due to violence within their population.

Cramped Area:

In tightly packed areas, the rate of which the female flies' mate with the males do not appear to change.However, the duration of which the flies are developed increases while the overall size of the flies reduces. Since the period of which the fly is conceived is elongated, it can be deduced that collectively the number of eggs produced is lower than that when they are not forced into a cramped area. Since the size of the flies and its general effect in the flies' wellbeing is unknown, there is a possibility that smaller flies mate or die at a different rate. Since this experiment is done under lab conditions, the area in which the flies are bred needs to remain the same so as to avoid any loss of flies when transporting the fly community in various stages of the experiment. Consequently, at a certain stage, the fly's population (which is increasing exponentially)will reach a point where the space (controlled incubation storage) will be too small for the population and hence affect the size, rate of death and rate of mating which in turn affects the results collected from the experiment.

Reproductive Factors:

The inaccuracy of the data produced can also be attributed to the imprecise assumption that the populations of female and male fruit flies are constant. Varying the data even slightly so that there are more males than females or more females than males can skew the data. For instance if there are more males than females or more females than males, less eggs will be produced because there are less male-female pairs reproducing in the span of one week. This assumption however also depends on the number of crossover pairings, one male fruit fly may mate with multiple females and one female may mate with multiple males. Female fruit flies also have the ability to store sperms for later use and therefore produce a larger number of eggs over their entire life span than the initially assumed 70 eggs (10 eggs every week for 7 weeks of adulthood life). Moreover, the number of eggs laid will vary biologically as well as the time of death for each fruit fly being different and possibly occurring during an earlier stage (such as pupa or larva).

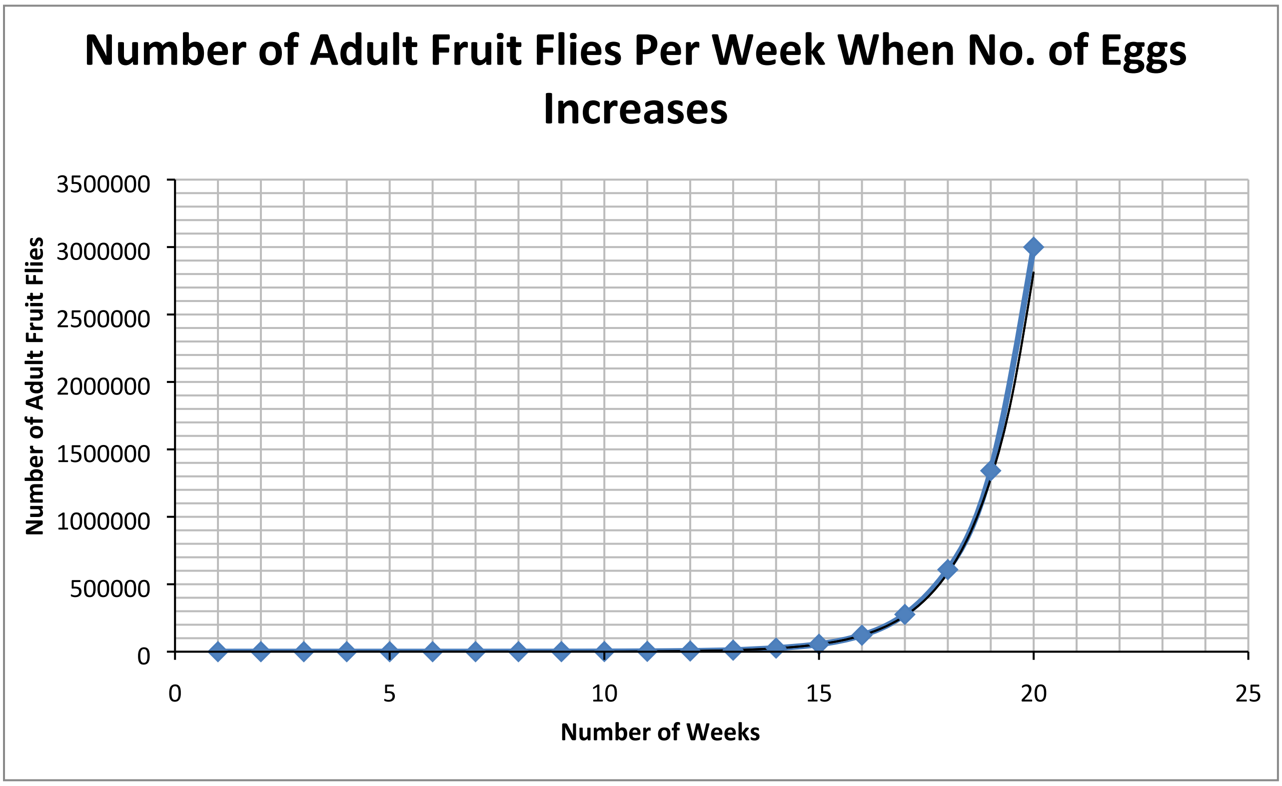
Calculations:

Although not all of these factors can be calculated mathematically, this investigation will explore the impact some of these factors may have on the original data.Mutation In Number Of Eggs Produced (More Eggs) Assumptions:

- More eggs are produced than the given assumption from the task.
- For every 5 additional pairs, an extra 10 eggs will be produced.
- This does not include deaths of adults every seventh week.

Mutation In No. Of Eggs Produced With An Addition Of A Constant Per Week			
Weeks	Eggs	Pupa	Adults
0	0	1M + 1F	0
1	0	0	1M + 1F
2	5M + 5F	0	1M + 1F
3	5M + 5F	5M + 5F	1M + 1F
4	5M + 5F	5M + 5F	6M + 6F
5	35M + 35F	5M + 5F	11M + 11F
6	65M + 65F	35M + 35F	16M + 16F
7	95M +95F	65M +65F	51M +51F
8	305M + 305F	95M +95F	116M + 116F
9	695M + 695F	305M + 305F	211M + 211F
10	1265M + 1265F	695M + 695F	516M + 516F
11	3095M + 3095F	1265M +1265F	1211M + 1211F
12	7265M + 7265F	3095M + 3095F	2476M + 2476F
13	14855M + 14855F	7265M + 7265F	5571M + 5571F
14	33425M +33425F	14855M + 14855F	12836M + 12836F
15	77015M + 77015F	33425M + 33425F	27691M + 27691F
16	166145M + 166145F	77015M + 77015F	61116M + 61116F

17	366695M + 366695F	166145M + 166145F	138131M + 138131F
18	828785M + 828785F	366695M + 366695F	304276M + 304276F
19	1825655M + 1825655F	828785M + 828785F	670971M + 670971F
20	4025825M + 40525825F	1825655M + 1825655F	1499756M + 1499756F



Mutation In Number Of Eggs Produced With Increasing Adult Population (Less):

In order to consider a more realistic scenario, the possibility of mutations and birth defects have been considered and investigated in order to investigate their effects of the general population of flies being studied. With the prompt provided, the number of eggs produced out of 10 pairs account to 100 eggs, however, with the mutation, for every 10 additional pairs 20 less eggs survive. Any deaths which are caused by the mutations are separate from those caused due to age.

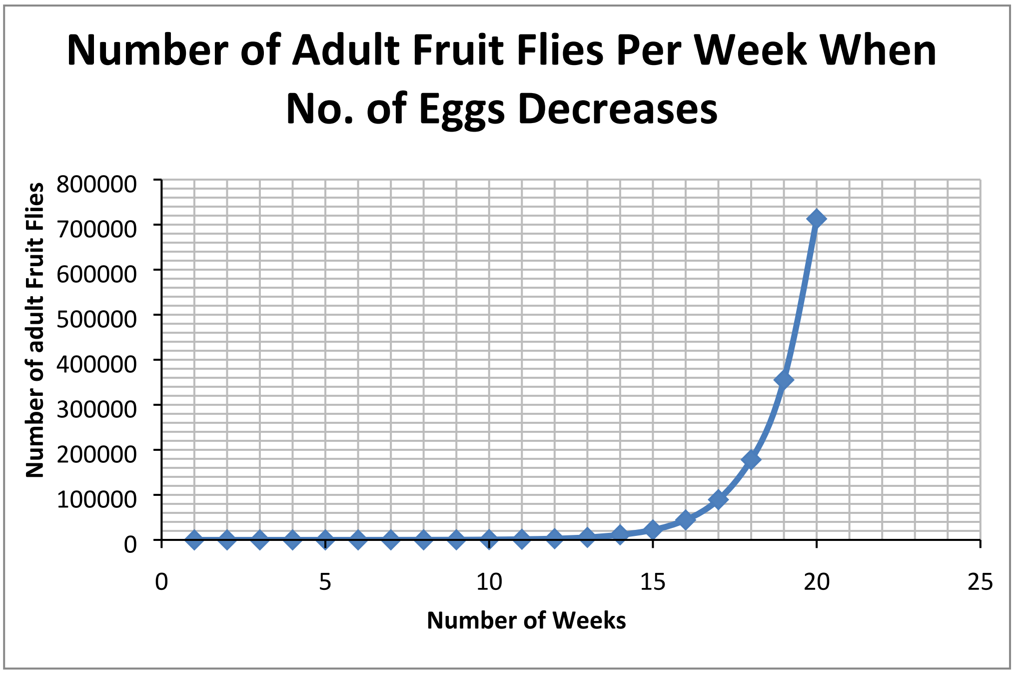
Through the trends, it is observed that the amount of pupas and adults alternatively overtake one another (i.e., in weeks 1, 2, 6, 9 etc the number of pupas in larger than that of adults while in weeks 3, 4, 5, 7, 8 etc the number of adults pass the number of pupas.). The wavering trends can be attributed to the mutations affecting a group of flies differently. For example, the first 20 flies would not be affected by the mutation as it is following the addition of 10 pairs, thus, for every 10 additional pair the number of eggs produced varies further from the initial. If the flies are considered to be in batches, it is evident that as 10 additional pairs are added, the number of eggs produced is reduced by 20 however, since the number of eggs produced by each batch is increasing, a single batch may be affected more times than that previous to it. The eggs then grow to become pupas and then to adults; which is why the inconsistent addition of the amount of pupas and adults result in an inconsistent distinction whether there would be more adults or pupas after a given time.

The following mutation has been mapped out in the following table:

Assumptions:

- Less eggs are produced.
- For every 10 additional pairs, 20 less eggs will be produced.
- This does not include deaths of adults every seventh week.

Mutation In Number Of Eggs Produced With Increasing Adult Population (Less)			
Weeks	Eggs	Pupa	Adults
0	0	1M + 1F	0
1	0	0	1M + 1F
2	5M + 5F	0	1M + 1F
3	5M + 5F	5M + 5F	1M + 1F
4	5M + 5F	5M + 5F	6M + 6F
5	30M + 30F	5M + 5F	11M + 11F
6	45M + 45F	30M + 30F	16M + 16F
7	70M + 70F	45M + 45F	46M + 46F
8	190M + 190F	70M + 70F	91M + 91F
9	365M + 365F	190M + 190F	161M + 161F
10	645M + 645F	365M + 365F	351M + 351F
11	1405M + 1405F	645M + 645F	716M + 716F
12	2870M + 2870F	1405M + 1405F	1361M + 1361F
13	5445M + 5445F	2870M + 2870F	2766M + 2766F
14	11070M + 11070F	5445M + 5445F	5636M + 5636F
15	22550M + 22550F	11070M + 11070F	11081M + 11081F
16	44325M + 44325F	22550M + 22550F	22151M + 22151F
17	88605M + 88605F	44325M + 44325F	44701M + 44701F
18	178805M + 178805F	88605M + 88605F	89026M + 89026F
19	356110M + 356110F	178805M + 178805F	177631M + 177631F
20	710525M + 710525F	356110M + 356110F	356436M + 356436F



Premature Death Of Eggs And Delayed Death Once Adults:

Realistically, there are many factors that would prevent a fly egg to progress and become a larvae, these factors may include but are not limited to: mutations, disease, etc. In order to investigate the effects of not having a full set of eggs, the following scenario is investigated; Assuming that adults had a life span of 20 weeks and 10% of all eggs produced die before becoming larvae (death being separate from flies that die due to old age). At every stage where by the total number of eggs are odd, there will always be more female and where the total number of eggs is even, they will be an equal amount of male and female eggs.

When there are an odd number of flies, it is decided that they become females because female flies grow up to become around 25% larger and thus in comparison to their male counterpart, they are more likely to survive past the egg stage and grow to become female flies as their size contribute to more resistance against slightly changing environments. The discrepancy between the two contributes to there being more female flies than that of males. (***APA in-text reference: Web.as.uky.edu. 2021. [online] Available at: <http://web.as.uky.edu/biology/faculty/cooper/Population%20dynamics%20examples%20with%20fruit%20flies/Sex-Determining%20Guide.pdf> [Accessed 9 July 2021].***).

Alternatively, we can also consider the scenario where 90% of adults die by the 8th week (1 week later than what is in the prompt) while the remaining 10% of the total die by their 10th week alive (3 weeks later than normal) while also having 20% of all eggs die prior to becoming an adult.

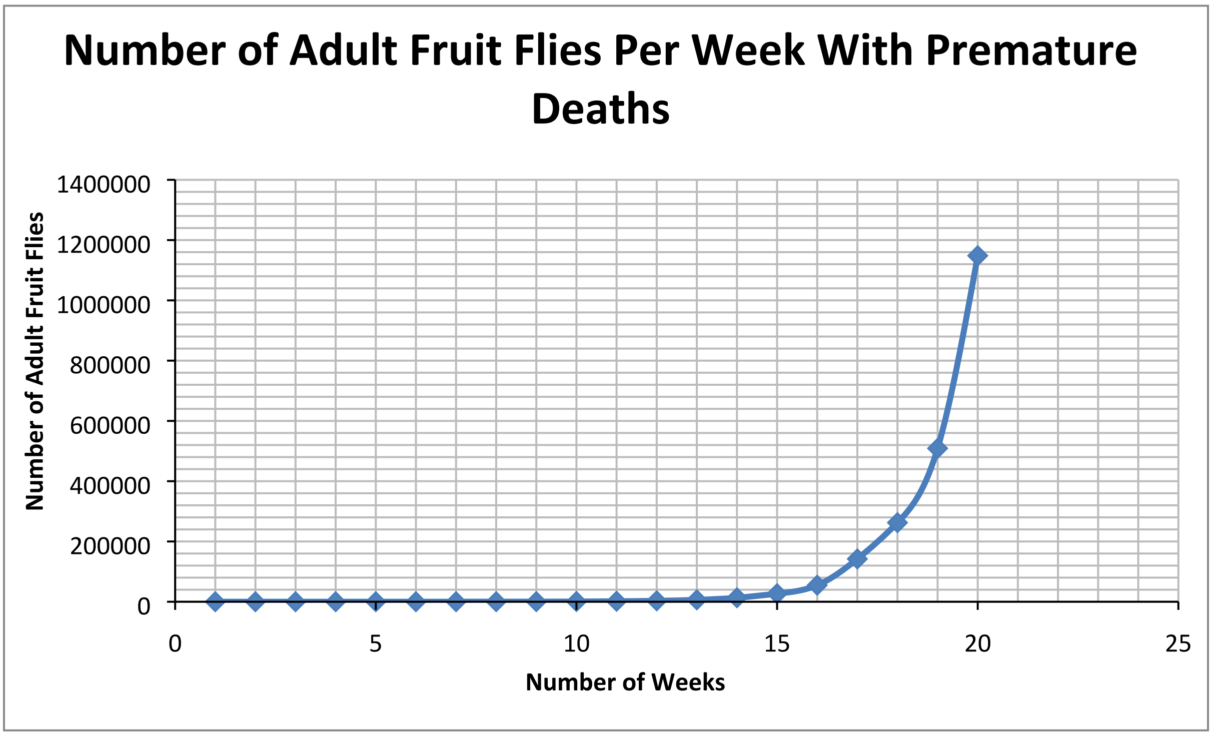
The table below shows this:

Assumptions:

- 10% of eggs produced die a premature death.
- At every odd number population of eggs there will be more females than males.
- This does not include deaths of adults every seventh week.

Premature Death Of Eggs.			
Weeks	Eggs	Pupa	Adults
0	0	1M + 1F	0
1	0	0	1M + 1F
2	5M + 5F	0	1M + 1F
3	5M + 5F	5M + 5F	1M + 1F
4	5M + 5F	5M + 5F	6M + 6F
5	27M + 27F	5M + 5F	11M + 11M
6	49M + 50F	27M + 27F	16M + 16F
7	72M + 72F	49M + 50F	43M + 43F
8	193M + 194F	72M + 72F	92M + 93F
9	414M + 414F	193M + 194F	164M + 165F
10	738M + 738F	414M + 414F	357M + 359F
11	1606M + 1607F	738M + 738F	771M + 773F
12	3469M + 3470F	1606M + 1607F	1509M + 1511F
13	6790M + 6791F	3469M + 3470F	3115M + 3118F
14	14017M + 14018F	6790M + 6791F	6584M + 6588F
15	29628M + 29628F	14017M + 14018F	13374M + 13379F

16	60183M + 60183F	43645M + 43645F	27391M + 27397F
17	123259M + 123260F	60183M + 60183F	71036M + 71042F
18	319662M + 319662F	123259M + 123259F	131219M + 131225F
19	590485M + 590486F	319662M + 319662F	254478M + 254484F
20	1145151M + 1145151F	590485M + 590485F	574140M + 574146F



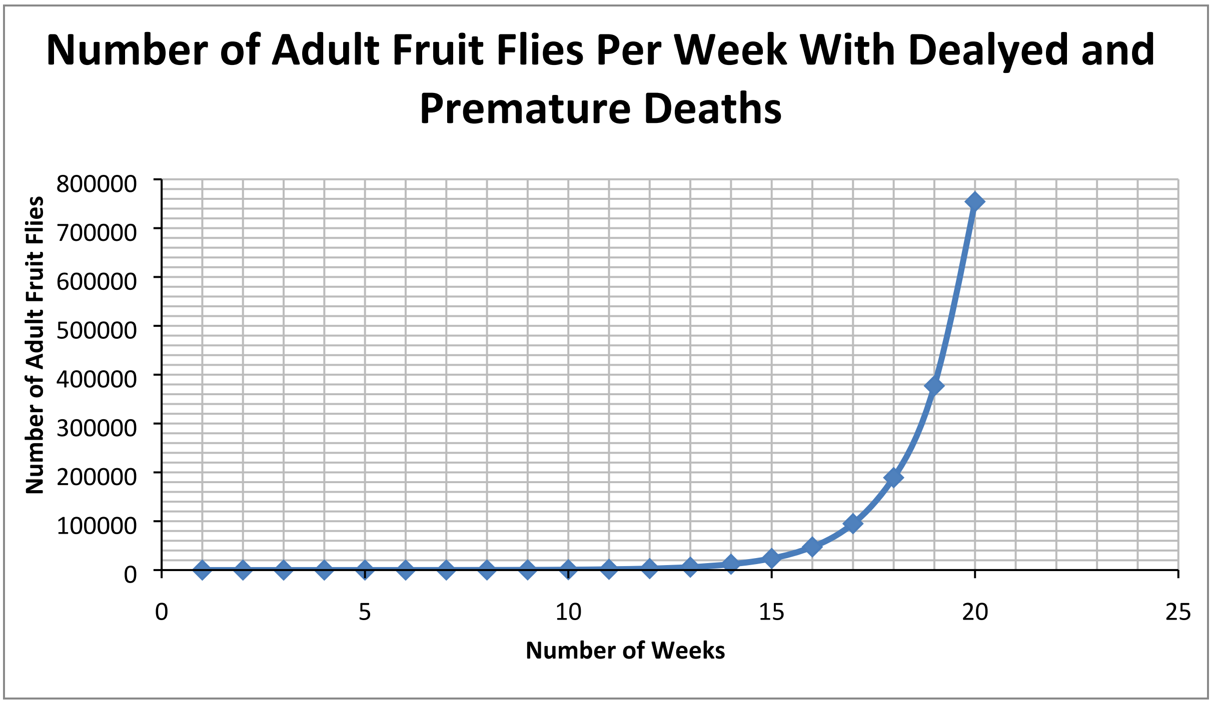
Delayed Deaths In The Adult Stage With Premature Deaths In Eggs Stage:

Assumptions:

- Death of adults normally on the 8th week.
- With delayed deaths, 10% of adult pairs will die on the 10th week instead; 90% of adult pairs will die on the 8th week.
- Additionally, 20 eggs will die after having lived a week, for every 100 eggs produced.
- Eg. at 230 eggs, only 230-2*20 eggs will become pupa.

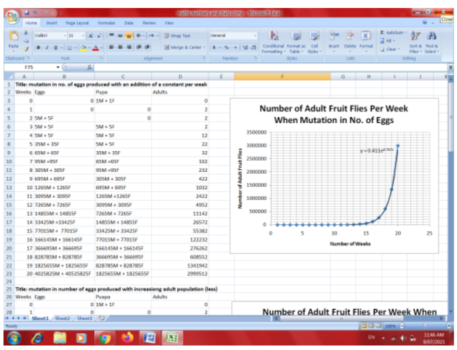
Delayed Deaths In The Adult Stage With Premature Deaths In Eggs Stage			
Weeks	Eggs	Pupa	Adults
0	0	1M + 1F	0
1	0	0	1M + 1F
2	5M + 5F	0	1M + 1F
3	5M + 5F	5M + 5F	1M + 1F
4	5M + 5F	5M + 5F	6M + 6F
5	30M + 30F	5M + 5F	11M + 11F
6	55M + 55F	30M + 30F	16M + 16F
7	80M + 80F	55M + 55F	46M + 46F
8	230M + 230F	80M + 80F	100M + 100F
9	500M + 500F	190M + 190F	180M + 180F

10	900M + 900F	400M + 400F	370M + 370F
11	1850M + 1850F	720M + 720F	765M +765F
12	3825M + 3825F	1490M + 1490F	1480M + 1480F
13	7400M + 7400F	3065M + 3065F	2965M + 2965F
14	14825M + 14825F	5920M + 5920F	6003M + 6003F
15	30015M + 30015F	11865M + 11865F	11870M + 11870F
16	59350M + 59350F	24015M + 24015F	23658M + 23658F
17	118290M + 118290F	47490M + 47490F	47494M + 47494F
18	237470M + 237470F	94650M + 94650F	94605M + 94605F
19	473025M + 473025F	189990M + 189990F	188567M + 188567F
20	942835M + 942835F	378425M + 378425F	377144M + 377144F



Technological Methods Used:

In creating this report and investigating the task question, the use of technology was applied. The use of websites and online software was required for research purposes, this includes websites such as GeoGebra and MathSolver. Microsoft Office Excel and Graphic calculators such as CAS were also useful tools that helped structure the data in neat tables and graphs.



Conclusion:

The aim of this investigation was to explore research questions related to a given scenario experimenting with the population growth of fruit flies. It can be concluded that theoretically, based on the given assumptions, the population growth of fruit flies should occur exponentially, with a differential equation acting as the relationship between the passage of time and the number of adult fruit flies present. The equation for the relationship between the number of weeks and fruit flies in other stages such as egg or pupa can also be identified and used to find the derivatives or solutions of the given equations. However, it can also be concluded from given research that the theoretical data is not representative of the population growth of fruit flies due to many acting factors that were not taken into account in the theoretical data.

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Appendix A:

*Epigenetics refers to the transfer of information through the addition of genetic markers unto an organism's DNA (deoxyribonucleic acid). The genetic marker is added due to environmental factors and can be reversible while others are transferred from parent to child. Epigenetics has been a medium of information sharing that surpasses the boundaries of death and allows for changes to be made unto the DNA so that the future generation can benefit from the experience of their ancestors. It is observed that through the genetic markers placed unto one's DNA, the evolutionary mutations which develops tend to be in favour of environment in which the flies presently live in, thus, the flies with the more suitable genome can survive longer and mate. The prevalence of genetic marker can be observed as evolutionary traits has been developed by the descendants which allow them to live better in climates that are not necessarily suited to promote mating.

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